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Mixed-Integer Nonconvex Quadratic Optimization Relaxations and Performance Analysis

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14. ABSTRACT

This project considers a class of general quadratic optimization problems involving both integer and continuous variables. These problems are strongly motivated by applications in optimal and dynamic resource management, cardinality constrained quadratic programs, and the matrix completion problems with non-convex regularity. The project addresses a fundamental question how to efficiently solve these problems, such as to find a provably high quality approximate solution or to fast find a local solution with probable structure. Given the non-convex nature of these problems, two relaxation approaches are considered: one is based on convex semidefinite relaxation (SDR), while the other is based on quasi-convex relaxations (QCR). In contrast to the classical mixed integer nonlinear programming approaches, no convexity is assumed for sub-problems when some integer variables are fixed. For the cardinality constrained QP problems, a QCR approach is proposed based on non-convex regularization. Compared to the known L1 relaxation, this relaxation gives better performance in practice. Below are selected highlight accomplishments made from the project.

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Final Report on AFOSR Grant FA9550-09-1-0306

October 8, 2016

This project considers a class of general quadratic optimization problems involving both integer and continuous variables. These problems are strongly motivated by applications in optimal and dynamic resource management, cardinality constrained quadratic programs, and the matrix completion problems with non-convex regularity. The project addresses a fundamental question how to efficiently solve these problems, such as to find a provably high quality approximate solution or to fast find a local solution with probable structure. Given the non-convex nature of these problems, two relaxation approaches are considered: one is based on convex (semidefinite) relaxation (CVR), while the other is based on quasi-convex relaxations (QCR). In contrast to the classical mixed integer nonlinear programming approaches, no convexity is assumed for sub-problems when some integer variables are fixed. For the cardinality constrained QP problems, a QCR approach is proposed based on non-convex regularization. Compared to the known L1 relaxation, this relaxation gives better performance in practice.

In this project, we like to do 1) Theoretical analysis of relaxations SDR and QCR, and 2) Develop efficient algorithms for solving large-scale relaxation problems, either finding a global or local solution.

Below are what we have achieved so far.

I. Convex Relaxation (CVS) Models and Analyses

"**A Dynamic Near-Optimal Algorithm for Online Linear Programming**" (Agrawal, Wang and Ye), Operations Research, 62(4) (2014) 876 - 890.

This paper considers a typical online 0-1 integer resource-allocation linear program - a natural optimization model that formulates many online resource allocation and revenue management problems is the online linear program (0-1-ILP) in which the constraint matrix is revealed column by column along with the corresponding objective coefficient. In such a model, a 0 or 1 decision variable has to be set each time a column is revealed without observing the future inputs and the goal is to maximize the overall objective function. In this paper, we provide a near-optimal algorithm for this general class of online problems under the assumption of random order of arrival and some mild conditions on the size of the ILP right-hand-side input. Specifically, our learning-based algorithm works by dynamically updating a threshold price vector of the LP relaxation at geometric time intervals, where the dual prices learned from the revealed columns in the previous period are used to determine the sequential decisions in the current period. Due to the feature of dynamic learning, the competitiveness of our algorithm improves over the past study of the same problem. We also present a worst-case example showing that the performance of our algorithm is near-optimal. This paper is already cited **74** times according to the Google Scholar.

“**Analytical results and efficient algorithm for optimal portfolio deleveraging with market impact**,” (Jingnan Chen, Liming Feng, Jiming Peng, Yinyu Ye), Operations Research, 62(1) (2014) 195-206.

In this paper we consider an optimal portfolio deleveraging problem, where the objective is to meet specified debt/equity requirements at the minimal execution cost. Permanent and temporary price impact is taken into account. With no restrictions on the relative magnitudes of permanent and temporary price impact, the optimal deleveraging problem reduces to a nonconvex quadratic program with quadratic and box constraints. Analytical results on the optimal deleveraging strategy are obtained. They provide guidance on how we liquidate a portfolio according to endogenous and exogenous factors. A Lagrangian relaxation method is proposed to solve the nonconvex quadratic program numerically. By studying the breakpoints of the Lagrangian problem, we obtain conditions under which the Lagrangian method returns an optimal solution of the deleveraging problem. When the Lagrangian algorithm returns a suboptimal approximation, we present upper bounds on the loss in equity caused by using such an approximation. This piece of work won the **Morgan Stanley 2012 Prize for Excellence in Financial Markets**, First runner-up.

“**Space tensor conic programming**,” (L Qi and Y Ye), Computational Optimization and Applications, 6(26) (2013) 1-13.

Space tensors appear in physics and mechanics, and they are real physical entities. Mathematically, they are tensors in the three-dimensional Euclidean space. In the research of diffusion magnetic resonance imaging, convex optimization problems are formed where higher order positive semi-definite space tensors are involved. In this short paper, we investigate these problems from the viewpoint of conic linear programming (CLP). We characterize the dual cone of the positive semi-definite space tensor cone, and study the CLP formulation and the duality of the positive semi-definite space tensor conic programming problem.

II. Quasi-Convex Relaxation (QCR) Models and Analyses

“**Complexity Analysis of Interior Point Algorithms for Non-Lipschitz and Nonconvex Minimization**,” (W. Bian, X. Chen, and Ye), Math Programming, 149 (2015) 301-327.

In this paper we propose a first order interior point algorithm for a class of non-Lipschitz and nonconvex minimization problems with box constraints, which arise from emerging applications in variable selection and regularized optimization. The objective functions of these problems are continuously differentiable typically at interior points of the feasible set. Our first order algorithm is easy to implement and the objective function value is reduced monotonically along the iteration points. We show that the worst-case iteration complexity for finding an ϵ scaled first order stationary point is $O(\epsilon-2)$. Furthermore, we develop a second order interior point algorithm using the Hessian matrix, and solve a quadratic program with a ball constraint at each iteration. Although the second order interior point algorithm costs more computational time than that of the first-order

algorithm in each iteration, its worst-case iteration complexity for finding an ϵ scaled second order stationary point is reduced to $O(\epsilon^{-3/2})$. Note that an ϵ scaled second order stationary point must also be an ϵ scaled first order stationary point. These results are the state of art in complexity analysis of non-convex optimization.

"**Complexity of Unconstrained L2-Lp Minimization," (Chen, Ge, Wang, Ye), Math Programming**, 143 (1-2) (2014) 371-383.

This paper resolved an important open question in cardinality constrained quadratic optimization and regression. Here we consider the unconstrained Lq-Lp minimization: find a minimizer of $\|Ax-b\|_q^q+\lambda\|x\|_p^p$ for given $A\in R^{m\times n}$, $b\in R^m$ and parameters $\lambda>0$, $p\in[0,1)$ and $q\geqslant 1$. This problem has been studied extensively in many areas. Especially, for the case when q=2, this problem is known as the L2−Lp minimization problem and has found its applications in variable selection problems and sparse least squares fitting for high dimensional data. Theoretical results show that the minimizers of the Lq-Lp problem have various attractive features due to the concavity and non-Lipschitzian property of the regularization function $\|\cdot\|_p^p$. In this paper, we show that the Lq-Lp minimization problem is strongly NP-hard for any $p\in[0,1)$ and $q\geqslant 1$, including its smoothed version. On the other hand, we show that, by choosing parameters (p,λ) carefully, a minimizer, global or local, will have certain desired sparsity. We believe that these results provide new theoretical insights to the studies and applications of the concave regularized optimization problems. The above two papers have been cited **71** times

"**Simultaneous Beam Sampling and Aperture Shape Optimization for Station Parameter Optimized Radiation Therapy (SPORT)**" (M Zarepisheh, Y Ye, S Boyd, R Li, L Xing), Medical Physics 41(6) (2014) 292-292.

Station parameter optimized radiation therapy (SPORT) was recently proposed to fully utilize the technical capability of emerging digital linear accelerators, in which the station parameters of a delivery system, such as aperture shape and weight, couch position/angle, gantry/collimator angle, can be optimized simultaneously. SPORT promises to deliver remarkable radiation dose distributions in an efficient manner, yet there exists no optimization algorithm for its implementation. The purpose of this work is to develop an algorithm to simultaneously optimize the beam sampling and aperture shapes.

In this paper we build a mixed integer quadratic programming (MIQP) model with the fundamental station point parameters as the decision variables. To solve the resulting large-scale optimization problem, the authors devise an effective algorithm by integrating three advanced optimization techniques: column generation, subgradient method, and pattern search. Column generation adds the most beneficial stations sequentially until the plan quality improvement saturates and provides a good starting point for the subsequent optimization. It also adds the new stations during the algorithm if beneficial. For each update resulted from column generation, the subgradient method improves the selected stations locally by reshaping the apertures and updating the beam angles toward a descent subgradient direction. The algorithm continues to improve the selected stations locally

and globally by a pattern search algorithm to explore the part of search space not reachable by the subgradient method. By combining these three techniques together, all plausible combinations of station parameters are searched efficiently to yield the optimal solution.

A SPORT optimization framework with seamlessly integration of three complementary algorithms, column generation, subgradient method, and pattern search, was established. The proposed technique was applied to two previously treated clinical cases: a head and neck and a prostate case. It significantly improved the target conformality and at the same time critical structure sparing compared with conventional intensity modulated radiation therapy (IMRT). In the head and neck case, for example, the average PTV coverage D99% for two PTVs, cord and brainstem max doses, and right parotid gland mean dose were improved, respectively, by about 7%, 37%, 12%, and 16%.

"**Folded Concave Penalized Sparse Linear Regression: Sparsity, Statistical Performance, and Algorithmic Theory for Local Solutions**," (H. Liu, T. Yao, R. Li, Y. Ye) manuscript, 2^{nd} revision in Math Programming (2016).

This paper concerns the folded concave penalized sparse linear regression (FCPSLR) problem, which is an alternative sparse recovery method than Lasso. Although FCPSLR yields desirable recovery performance when solved globally, computing a global solution is NP-complete. Despite some existing statistical performance analyses on local minimizers or on specific FCPSLR-based learning algorithms, it still remains an open question whether local solutions that are known to admit fully polynomial-time approximation schemes (FPTAS) may already be sufficient to ensure the statistical performance, and whether that statistical performance can be non-contingent on the specific designs of computing procedures. Seeking to address this question, this paper presents the following three-fold results: (i) Any local solution (stationary point) is a sparse estimator, under some conditions on the parameters of the folded concave penalties. (ii) Perhaps more importantly, any local solution satisfying a significant subspace second-order necessary condition (S3ONC), which is weaker than the second-order KKT condition, yields a bounded error in approximating the true parameter with high probability. In addition, if the minimal signal strength is sufficient, the S3ONC solution likely recovers the oracle solution. (iii) We apply (ii) to the special case of FCPSLR with minimax concave penalty (MCP) and show that under the restricted eigenvalue condition, any S3ONC solution with a better objective value than the Lasso solution entails the strong oracle property. We also show that to guarantee the S3ONC admits the FPTAS algorithm.

III. Fast Algorithm Development

"**The simplex method is strongly polynomial for deterministic Markov decision processes**," (Ian Post and Ye), Math of Operations Research, 40 (4) (2015) 859-868.

The simplex method continues to be the major method for integer linear programming, but its complexity analyses are most negative. In this paper we finally prove that the

simplex method with the highest gain/most-negative-reduced cost pivoting rule converges in strongly polynomial time for deterministic Markov decision processes (MDPs) regardless of the discount factor. For a deterministic MDP with n states and m actions, we prove the simplex method runs in O(n^3 m^2 log^2 n) iterations if the discount factor is uniform and O(n^5 m^3 log^2 n) iterations if each action has a distinct discount factor. Previously the simplex method was known to run in polynomial time only for discounted MDPs where the discount was bounded away from 1. This piece of work won **the 2013 Second Prize of INFORMS Nicholson Student Paper Competition**.

"**The Direct Extension of ADMM for Multi-block Convex Minimization Problems is Not Necessarily Convergent**," (C. Chen, B. He, Y. Ye, X. Yuan), Math Programming, 155(1-2) (2016) 57-79.

The alternating direction method of multipliers (ADMM) is now widely used in many fields, and its convergence was proved when two blocks of variables are alternatively updated. It is strongly desirable and practically valuable to extend the ADMM directly to the case of a multi-block convex minimization problem where its objective function is the sum of more than two separable convex functions. However, the convergence of this extension has been missing for a long time—neither an affirmative convergence proof nor an example showing its divergence is known in the literature. In this paper we give a negative answer to this long-standing open question: The direct extension of ADMM is not necessarily convergent. We also present a sufficient condition to ensure the convergence of the direct extension of ADMM, and give an example to show its divergence. This paper is already cited **147** times according to the Google Scholar.

"**On the Expected Convergence of Randomly Permuted ADMM**," (Sun, LUO and Ye), in OPT2015 (Optimization@NIPS for Machine Learning), 2015.

The alternating direction method of multipliers (ADMM) is now widely used in many fields, and its convergence was proved when two blocks of variables are alternately updated. It is computationally beneficial to extend the ADMM directly to the case of a multi-block convex minimization problem. Unfortunately, such an extension fails to converge even when solving a simple square system of linear equations. In this paper, however, we prove that, if in each step one randomly and independently permutes the updating order of any given number of blocks followed by the regular multiplier update, the method will converge in expectation for solving the square system of linear equations. Our result indicates that for ADMM the cyclic update rule and the random permutation update rule are fundamentally different. To the best of our knowledge, this is the first example that such a difference is rigorously established for a specific optimization algorithm, although the random permutation rule has been reported to be experimentally better than the cyclic rule in many large-scale optimization problems. Our analysis technique of random permutation might be useful in other contexts.

“**ALGORITHMS FOR UNSYMMETRIC CONE OPTIMIZATION AND AN IMPLEMENTATION FOR PROBLEMS WITH THE EXPONENTIAL CONE**,” Ph.D. Thesis, Santiago Akle Serrano, Stanford, 2015. The thesis was founded by the project, and it produced a fast solver for un-symmetric conic optimization solver included in an **open source code** http://web.stanford.edu/~boyd/papers/ecos.html

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Abstract

This project considers a class of general quadratic optimization problems involving both integer and continuous variables. These problems are strongly motivated by applications in optimal and dynamic resource management, cardinality constrained quadratic programs, and the matrix completion problems with non-convex regularity. The project addresses a fundamental question how to efficiently solve these problems, such as to find a provably high quality approximate solution or to fast find a local solution with probable structure. Given the non-convex nature of these problems, two relaxation approaches are considered: one is based on convex semidefinite relaxation (SDR), while the other is based on quasi-convex relaxations (QCR). In contrast to the classical mixed integer nonlinear programming approaches, no convexity is assumed for sub-problems when some integer variables are fixed. For the cardinality constrained QP problems, a QCR approach is proposed based on non-convex regularization. Compared to the known L1 relaxation, this relaxation gives better performance in practice. Below are selected highlight accomplishments made from the project.
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for solving typical online 0-1 integer linear program - a natural optimization model that formulates many online resource allocation problems. The paper has been cited 74 times (Google Scholar).
2)Complexity of unconstrained L_2-L_p minimization (X Chen, D Ge, Z Wang, Y Ye), Mathematical Programming 143 (1-2) (2014) 371-383; and "Complexity Analysis of Interior Point Algorithms for Non-Lipschitz and Nonconvex Minimization," (W. Bian, X. Chen, and Ye), Math Programming, 149 (2015) 301-327. In this papers we propose first-order and second-order interior point algorithm for a class of non-Lipschitz and nonconvex strongly NP-hard minimization problems with box constraints, which arise from emerging applications in variable selection and regularized optimization. The complexity result of the algorithms represent the state of the art in the field. The two paper is cited 71 times.
3)"The simplex method is strongly polynomial for deterministic Markov decision processes," (Ian Post and Ye), Math of Operations Research 40 (4) (2015) 859-868. The simplex method continues to be the major method for integer linear programming, but its complexity analyses are most negative. In this paper we finally prove that the simplex method with the highest gain/most-negative-reduced cost pivoting rule converges in strongly polynomial time for deterministic Markov decision processes (MDPs) regardless of the discount factor. This piece of work won the 2013 Second Prize of INFORMS Nicholson Student Paper Competition.
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